Smarting up water distribution networks with an entropy-based optimal sensor placement strategy

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Abstract: Presented herein is a proposed greedy-search sensor placement optimization heuristic for the detection of water leaks in water distribution networks (WDN). The proposed method is based on entropy, a measure of uncertainty about the source of information, and its main mathematical properties of maximality, subadditivity and equivocation. The method proposes an entropic metric which is subsequently utilized in selecting nodal locations and in heuristically searching for the locations that maximize the total entropy in the WDN, relating maximal entropy with maximal sensing coverage.

Keywords: water distribution networks, leak detection, sensor placement

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1. Introduction

As existing water distribution networks (WDN) age and their deterioration accelerates, their constituent parts (mainly pipes) are increasingly at risk of failure. In fact, each year hundreds of kilometers of pipes across the globe are upgraded or replaced in an attempt to reduce water loss due to pipe bursts. Water loss, defined as the percentage of drinking water placed into a WDN that does not find its way to billed customers (or unbilled authorized users), consists of two broad classes: apparent losses and real losses. The former category refers to the non-physical losses that occur in utility operations, i.e., this is water that is consumed but is not properly measured, accounted for or paid for. The latter category refers to the physical losses of water from the distribution system, including pipe breaks and leaks.

Research to-date has helped identify a number of potential time-invariant and time-dependent risk factors contributing to pipe breaks. Among them include factors such as a pipe’s age, diameter and material, as well as the network’s operating pressure and water flow. Several findings on risk assessment and prioritization of “repair-or-replace” actions were also reported by Christodoulou et al. based on neuro-fuzzy systems, survival analysis and geospatial clustering of WDNs under both normal and abnormal operating conditions. Their findings reinforced the need for real-time monitoring of a WDN’s key operating parameters (water pressure and flow), and also of soil moisture and acoustic signals within the WDN (as such signals may relate to leaking water).

Real-time monitoring is nowadays essentially performed by supervisory control and data acquisition systems (SCADA) and by use of sensors strategically located across the network. The goal in placing such sensors is to maximize their sensing effectiveness while also limiting their deployment and operational costs. Within such a framework of real-time sustaina-
ble management of WDN and water loss detection, the issue of sensor-placement optimization is of high importance.

2. State of Knowledge on Sensor Placement

As aforementioned, sensor placement optimization is a task critical to water loss detection. Unlike monitoring spatial phenomena such as temperature, humidity, noise or pollution, where sensors act radially, sensing in piping networks is primarily restricted to longitudinal actions (e.g., water flow and pressure, acoustic signals).

In the case of water loss detection by use of spatial monitoring (e.g., sensing soil moisture) one approach is to assume that sensors have a fixed sensing radius and then to solve the task graphically, or by use of GIS-based spatial analysis, or, as an instance of the art-gallery problem, or by use of Voronoi diagrams. This assumption, though, of radially-fixed sensing is erroneous for it fails to take into consideration that (i) a sensor’s sensing capability is not radially-invariant, and (ii) signal correlations are not always characterized by radial geometries especially when more than one sensor is needed to localize a signal. In the case of Voronoi diagrams, the method is also biased to the density of points (i.e., possible locations for sensors) and unreliable circumstantially of the plane in study.

An alternative approach is to treat sensing not as a spatial field but rather as a probabilistic data field, assuming that its efficiency can be modeled by a multivariate normal distribution (i.e., a Gaussian Process, “GP”, model). In the GP-model approach, data from a pilot study or expert knowledge is used to learn the parameters of the underlying GP distribution and then the learned GP model is used to forecast data on the same distribution. Thus, one can model the effect of placing sensors at particular locations by use of GP models and thus optimize the sensors’ positions.

An alternative optimization criterion was proposed by Caselton and Zidek, based on the concept of ‘mutual information’ which seeks sensor placements that are most informative about unsensed locations. The same criterion, coupled with a combinatorial optimization problem, was also employed by Krause et al. who proposed an algorithm which combinatorially selects k out of n possible sensor locations by first utilizing a lazy evaluation technique that exploits submodularity to reduce significantly the number of sensor locations that need to be checked, and then reducing the order of computational complexity by exploiting locality in sensing areas.

As Krause et al. reported, many criteria had been proposed for characterizing the quality of placements given a GP model, including placing sensors at the points of highest entropy (variance) in the GP model. A typical sensor placement technique is to greedily add sensors where uncertainty about the phenomena is highest. A similar approach related to entropy and mutual information was also reported by Guestrin et al. who suggested that sensors should be placed so as to maximize mutual information (i.e., maximum joint entropy), and used a greedy-variance heuristic as an approximation to the problem. Further, in order to address the problem of sensors being placed far apart along the boundary and information being ‘wasted’, they proposed a weighting heuristic. Unfortunately, though, as Krause et al. and Guestrin et al. reported, “this criterion suffers from a significant flaw: entropy is an indirect criterion, not considering the prediction quality of the selected placements. The highest entropy set, that is, the sensors that are most uncertain about each other’s measurements, is usually characterized by sensor locations that are as far as possible from each other. Thus, the entropy criterion tends to place sensors along the borders of the area of interest... Since a sensor usually provides information about the area around it, a sensor on the boundary ‘wastes’ sensed information”.

Entropy-related work on sensor placement optimization was also discussed by Chung et al. and Yang et al. The former investigated the problem of determining optimal pressure monitoring locations and proposed a method based on entropy, defining entropy as the amount of information calculated from the pressure change due to the variation of discharge. Their method required the use of hydraulic (EPANET) models for the investigation of the effect of abnormal conditions on the entire network (pressure changes) and the optimal locations for pressure sensors were selected to be the nodes having the maximum information from other nodes. The latter presented a feature extraction and leak detection system using approximate entropy to discriminate the leak signal from the non-leak acoustic sources.

Dorini et al., in addressing the problem of early detection of water contamination, formulated an optimal sensor placement methodology as a constrained multi-objective optimization problem and solved it based on the Noisy Cross-Entropy Sensor Locator (nCESL) algorithm. Contemporary work by several
researchers\textsuperscript{[9,17–19]} also addressed the same problem, offering a number of possible solution alternatives.

Aral et al.\textsuperscript{[20]} provided a methodology based on simulation and a single-objective function approach which incorporates multiple factors used in the design of a system, with a progressive genetic algorithm being used for the solution of the model. Genetic algorithms were also the focus of Preis and Ostfeld\textsuperscript{[21]} who presented a modified genetic algorithm scheme for contaminant source characterization using three types of perfect and imperfect sensors.

Diwold et al.\textsuperscript{[22]} presented a population-based ant colony optimization algorithm for sensor placement in water networks. Ant colony optimization was also the research subject of Afshar and Marino\textsuperscript{[23]} who presented a numerical procedure for the optimization of the position of water quality monitoring stations in a pressurized water distribution system. The procedure, which is based on the choice of the set of sampling stations which maximizes the monitored volume of water while keeping the number of stations at a minimum, is formulated in terms of integer programming and its solution approximated by means of a multi-objective multi-colony ant algorithm.

A number of mathematical programming approaches are also presented in literature. Berry et al.\textsuperscript{[24]} present a mixed-integer programming formulation for sensor placement optimization in municipal water distribution systems that includes the temporal characteristics of contamination events and their impacts. The information is utilized in computing the impact of a contamination event over time and determining affected locations, by quantifying the benefits of sensing contamination at different junctions in the network. Berger-Wolf et al.\textsuperscript{[25]} considered two variants of sensor placement for contamination detection and showed that the sensor and time constrained versions of the problem are polynomially equivalent. Carr et al.\textsuperscript{[26]} presented a series of related robust optimization models for placing sensors in municipal water networks to detect contaminants that are maliciously or accidentally injected. The sensor placement problem is formulated as a mixed-integer programming problem, for which the objective coefficients are not known with certainty. They then consider a restricted absolute robustness criterion that is motivated by natural restrictions on the uncertain data, and define three robust optimization models that differ in how the coefficients in the objective vary. Watson et al.\textsuperscript{[27]} also presented mixed-integer linear programming models for the sensor placement problem over a range of design objectives. Using two real-world water systems, they showed that optimal solutions with respect to one design objective are typically highly sub-optimal with respect to other design objectives. The implication is that robust algorithms for the sensor placement problem must carefully and simultaneously consider multiple, disparate design objectives.

3. Entropy — Overview and Relevant Properties

Entropy ($H_x$), in physics, is a measure of the unavailability of a system’s energy to do work and, by extent, a measure of the smoothness with which a transformation occurs and of the disorder and the amount of wasted energy during the transformation from one state to another. Mathematically, entropy can be expressed as the product of the probability mass function ($p_x$) of a variable $x$, times the natural logarithm of the inverse of the probability (Equation 1).

$$H_x = \sum_{x} p_x \ln \left( \frac{1}{p_x} \right)$$

Among entropy’s principal properties, three are of particular importance: subadditivity, maximality and equivocation.

- **Subadditivity** denotes that a function’s value for the sum of two elements is always less than or equal to the sum of the function’s values for each element.
- **Maximality** states that the entropy function, $H(p_1, p_2, \ldots, p_n)$, takes the greatest value when all admissible outcomes have equal probabilities ($p_1 = p_2 = \ldots = p_n$). In other words, maximal uncertainty is reached for the equiprobability distribution of possible outcomes.
- **Equivocation** is in effect the conditional entropy of one random variable against another, and it quantifies the remaining entropy (i.e., the uncertainty) of a random variable $Y$ given that the value of another random variable $X$ is known.

4. A Closer Look at Equivocation

In mathematical terms, equivocation is referred to as the entropy of $Y$ conditional on $X$. It is written as $H(Y \mid X)$ and can be shown to be governed by the following equation:

$$H(Y \mid X) = \sum_{x \in X} [p(x)H(Y \mid X = x)]$$

$$= \sum_{x \in X} \sum_{y \in X} p(x) \ln \frac{p(y \mid x)}{p(y \mid x)}$$

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\[ H_{XY} = \sum_{x \in X, y \in Y} p(x,y) \ln \frac{p(x,y)}{p(x)p(y)} \]  

(4)

It should be noted that the conditional entropy of \( Y \) given \( X \), \( H(Y | X) \), is bound by the entropy of \( Y \) and that the joint entropy of \( Y \) and \( X \), \( H(Y,X) \), is bound by the sum of the conditional entropies of \( H(Y | X) \) and \( H(X | Y) \).

\[ H(X | Y) \leq H(X) \]  

(5)

\[ H(Y,X) = H(X | Y) + H(Y | X) + I(X,Y) \]  

(6)

\[ I(X,Y) \leq H(X) \]  

(7)

where \( I(X,Y) \) is the mutual information between \( X \) and \( Y \). For independent \( X \) and \( Y \),

\[ H(Y | X) = H(Y) \quad \text{and} \quad H(X | Y) = H(X) \]  

(8)

As a corollary, the chain rule for conditional probability forms to be

\[ H(Y | X) = \sum_{x \in X, y \in Y} p(x,y) \ln \frac{p(x)}{p(x,y)} \]  

(9)

\[ = - \sum_{x \in X, y \in Y} \{p(x,y)\ln[p(x,y)]\} + \sum_{x \in X} \{p(x)\ln[p(x)]\} \]  

(10)

\[ = H(X,Y) + \sum_{x \in X} \{p(x)\ln[p(x)]\} \]  

(11)

\[ = H(X,Y) - H(X) \]  

(12)

An illustrative example of the above properties can be found in Christodoulou et al. [28].

5. Sensor Placement Optimization and Entropy Maximization

Since entropy is considered to be a good measure of a system’s order and stability, maximizing in value when a system is at an “equiprobability” state, then a higher degree of entropy should also indicate a more balanced system in terms of sensed information; one in which the information generated and/or distributed among its parts are of equal value. The sensor-placement optimization problem could thus be restated as one in which sensor locations are sought so that the system entropy is maximized.

Should one refer to the entropy equation (Equation 1) and define the probability term \( p_i \) in terms of a statistical measure of the ratio of a sensor’s sensing radius over the total length of the network, then the total network entropy, \( H_r \), for a single-type sensor would become

\[ H_T = \sum_{i=1}^{n_t} \left[ \frac{r_i}{L_T} \ln \left( \frac{1}{r_i / L_T} \right) \right] = -\sum_{i=1}^{n_t} \left[ \frac{r_i}{L_T} \ln \left( \frac{r_i}{L_T} \right) \right] \]  

(13)

where \( r_i \) is the sensing radius of sensor \( i \), \( n_t \) is the total number of sensors in the network, and \( L_T \) is the total length of the network.

It should be noted, though, that even though the above definition of \( p_s = r / L_T \) conforms to classical probability properties when sensors cover the entire network length, it does not conform when they do not and thus it is not mathematically correct. A solution to the problem, also conforming to the properties of entropy, is the definition of the ratio of \( r / L \) based on the arc length and not the network length. Thus, the value of \( p_s \) is taken to be the ratio of the sensor’s radius over the length of the network arc being sensed, and the total system entropy can be computed by summing up the entropy values for each arc. This definition also adheres to the fact that sensors at junctions of multiple arcs contribute to the entropy levels of these arcs, and helps avoid clustering of sensors at only a few parts of the network. Further, in order to account for the overlap in sensing radii of sensors placed at the end-nodes of an arc, and/or segment lengths shorter than the sensor’s sensing radius, the value of \( r_i \) used in Equation 13 is taken as the minimum between the segment length, \( L_i \), and the sensor’s sensing radius, \( x_i \) (in the case of one sensor), or the combined sensing radii (in the case of two sensors).

\[ r_i = \min \{ x_i; L_i \} \]  

(14)

Thus, for a single-type sensor,

\[ H_T = \sum_{i=1}^{n_t} \left[ \frac{n_i}{L_i} \ln \left( \frac{1}{r_i / L_i} \right) \right] = -\sum_{i=1}^{n_t} \left[ \frac{n_i}{L_i} \ln \left( \frac{n_i}{L_i} \right) \right] \]  

(15)

and for multiple sensor types, the total network entropy can similarly be defined as

\[ H_T = \sum_{j=1}^{n_r} \sum_{i=1}^{n_{r,j}} \left[ \frac{n_{r,j}}{P_{r,j}} \ln \left( \frac{n_{r,j}}{P_{r,j}} \right) \right] \]  

(16)

where \( j \) is the sensor-type index; \( n_r \) is the number of different sensor types used in the project; \( r_{i,j} \) is the number of units of sensor type \( j \) used on node \( i \); \( n_i \) is the total number of sensors in the network and \( r_{T,j} \) is the total number of units of sensor type \( j \) used in the network. The goal, therefore, is to maximize the network entropy subject to an allowable maximum number of sensors (of a specified sensing radius) or, equivalently, to maximize the entropy while minimizing the number of sensors used.
For example, in the sample arc shown in Figure 1(A) with length greater than the sum of the sensing radii, suppose we denote a sensed node by a filled circle and a node without a sensor as an empty circle, then based on the above definition of entropy the total entropy produced by the shown arc configuration can be computed as

\[
- \min \left( \frac{2x}{L} \right) \times \ln \left( \frac{2x}{L} \right) = -2 \times \ln \left( \frac{L}{L} \right).
\]

In the case of an arc length shorter than the sum of the sensing radii of the two nodal sensors (Figure 1(B)) the total pipe entropy is taken as

\[
- \min \left( \frac{2x}{L} \right) \times \ln \left( \frac{2x}{L} \right) = -L \times \ln \left( \frac{L}{L} \right) = 0.
\]

Figure 1. Proposed entropy-based sensor placement method: Generic cases of arc length (A) smaller and (B) greater than the sensing radii of sensors at the end-nodes.

Furthermore, for pipes whose length is smaller than the sum of the sensing radii of two nodal sensors (one at each pipe end), the entropy approach results in zero entropy values. If only one sensor is used (at either of the nodes), the entropy value is higher than zero, thus the entropy-maximization approach gives higher preference to a single-node arrangement compared to the two-node arrangement. A numeric demonstration can be seen in Figure 2. Suppose, for a pipe of length 300 meters, one sensor be used (at node \( n_i \)) with an assumed sensing radius of 200 meters, then the entropy for pipe \((n_i, n_j)\) is computed to be

\[
- \frac{200}{300} \times \ln \left( \frac{200}{300} \right) = 0.270.
\]

If two sensors are used (at nodes \( n_i \) and \( n_j \)) then the entropy is computed to be

\[
- \frac{300}{300} \times \ln \left( \frac{300}{300} \right) = 0.000.
\]

It should be noted that the method shows preference for a single sensor compared to sensors at both end-nodes, and that the proposed entropy-maximization approach holds true in more complicated sensor arrangements as well\[^{28}\].

6. Case Study of a Pipe Network

Let us now consider a more complicated piping network example, based on a real-life district metered area (DMA) network (Figure 3). The network consists of 448 nodes (possible sensor locations) and 633 pipes of varying length (in meters). The topology and nodal connectivity of the case study network are as shown in Figures 4 and 5, respectively.

Figure 3. Case-study network (shown in black outline), based on real-life DMA from Limassol’s (Cyprus) UWDN.

The network’s total entropy is the sum of all arc entropies as defined in the previous section, and can mathematically be expressed as

\[
H^\text{SYSTEM}_{i} = \sum_{i=1}^{n_i} H_{i,j} = \sum_{i=1}^{n_i} \left( H_{20,44} + H_{20,31} + H_{44,12} + H_{44,42} + H_{31,32} + \cdots + H_{386,385} + H_{384,385} \right)
\]
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Figure 4. Nodal connectivity of the DMA network considered.

Figure 5. Topology of the DMA network considered, with pipe lengths shown on network arcs.
where, \( i, j \) are connected arc nodes and \( n_t \) is the total number of nodes in the network. The equation essentially loops over all arcs in the network (from left to right in this case), identifying the connected nodes and calculating the resulting arc entropy, before summing up the total network entropy. This total entropy is zero when no sensors are present in the network (no-sensor configuration case) and can be evaluated to be \( H_t^{\text{ALL}} = 0.214 \) when sensors are located at each and every node (all-sensor configuration case). The all-sensor case is in effect sensitive to only the arcs whose length is greater than the addition of the sensing radii for each arc.

Let us now consider the entropy-maximization approach. The method starts with the nodal entropy values from the all-sensor configuration as the calculation base, and assumes that the entropy contributions to the total network entropy from the nodal sensors are not subject to the equivocation property. Upon ranking the nodal entropies in descending order, the method selects the node that contributes the maximum to the network entropy and places a sensor at that node. For the case-study network, the node to first receive a sensor is node “329” (Figure 6), causing an increase in entropy by

\[
- \frac{\min(47;100)}{47} \times \ln \frac{\min(47;100)}{47} = 0.000,
\]

and

\[
- \frac{\min(128;100)}{128} \times \ln \frac{\min(128;100)}{128} = -1.606
\]

along arcs [328,329], [330,329] and [384,329] respectively.

Upon assigning a sensor at a node, the entropy values of the connecting nodes are adjusted, considering equivocation and the entropy-maximization approach (Equations 14 and 15). For example, having placed a sensor at node “329” and thus sensing pipes [329,328], [329,330] and [329,384], the entropy values of connected nodes “328”, “330” and “384” need to be adjusted so that the equivocation property applies. Given that the total entropy of arcs [328,329], [330,329] and [384,329] when sensors are installed at both arc nodes is

\[
- \frac{\min(47;200)}{47} \times \ln \frac{\min(47;200)}{47} = 0.000,
\]

and

\[
- \frac{\min(128;200)}{128} \times \ln \frac{\min(128;200)}{128} = 0.000
\]

respectively, then the entropy adjustments on nodes “328”, “330” and “384” are 0.000, 1.606 and 0.317 respectively.

The revised nodal entropies are re-calculated and the node with the highest entropy is selected for sensor placement. The process is repeated until the maximum available number of sensors is reached. If, in this case-study network, we assume that a constraint on the number of sensors (\( n_t \)) is imposed of \( n_t = 6 \), then the proposed entropy-maximization approach arrives at the sensor topology shown in Figure 7, with the total network entropy calculated at \( H_T = 20.321 \).

The process may continue until the entropy heuristic places enough sensors in the WDN in study to cover it entirely. Solution of this problem is reached in short computational times (depending on the size of the network and the presumed sensing radius), especially if hydraulic parameters are not included in the analysis and the WDN is treated as static geometry of arcs and nodes. For the case study WDN, the number of sensors required for complete coverage is computed by the entropy heuristic to be 49.

7. Considering Hydraulics

As shown in Figure 7, the entropy-optimization heuristic moves towards maximality, not when all nodes are used as sensor positions, but rather when sensors are positioned at selected nodal positions (not necessarily at the boundaries of the network). However, as implemented above, the proposed method does not take into consideration the hydraulics in the network. Thus, it is more suitable to sensors which are hydraulic-invariant, such as acoustic sensors.

In the case of sensors which are directly affected by the hydraulics in the studied network (such as pressure or water flow sensors), one needs to also consider these hydraulic parameters in the entropy-maximization method, without having to evaluate the hydraulic model at each iteration of the greedy-search heuristic. The proposed algorithm could then be adjusted so as to allow for a weighing or for a spatial clustering mechanism that would give priority to specific areas in the network and steer nodal selections towards these areas.

Consider, for example, a terrain elevation map for the network area in study (as shown in Figure 8) and
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Figure 6. Entropy-based sensor placement optimization (sensor 1).

Figure 7. Entropy-based sensor placement optimization (first 6 sensors).
the assumption that the studied water distribution network is gravity-based. Thus, the hydraulics are greatly dependent on the elevations, with pipes at higher elevations operating at lower water pressures. Consider, further, that the network is divided in two elevation clusters, namely [22 m, 40 m) and [40 m, 60 m], and the need to locate a sensor in the [22 m, 40 m) zone having adequately covered the [40 m, 60 m) zone with the first 6 sensors positioned in the network (Figure 8).

The greedy-search entropy-maximization heuristic would now give precedence to nodes within the [22 m, 40 m) zone, identifying nodes “57” and “511” as the next ones to receive a sensor and bypassing three other nodes in the [40 m, 60 m] zone which were next in line to receive a sensor. The resulting entropy-based sensor placement configuration is as shown in Figure 9, with the latter two sensors shown in different color.

Even though the aforementioned process is static in nature and thus a rough approximation to the inclusion of water pressures in the analysis, a similar methodology would be employed in the presence of a hydraulic model forecasting nodal pressures based on consumer demand (and not gravity). An initial hydraulic estimation of the pressures would facilitate the creation of a pressure map for the network which would then be used to spatially identify the pressure clusters in the network. These pressure zones would then be used as a weighing mechanism for steering the location of pressure sensors in each of the areas of interest.

8. Conclusion

The work presented herein discusses sensor placement optimization for water distribution networks, proposing a greedy-search heuristic based on entropy and its subadditivity, maximality and equivocation properties. The proposed method formulates the sensor placement optimization problem as an entropy-maximization problem, with entropy defined in terms of the ratio of a sensor’s sensing distance over the length of the pipe being monitored, and its sum over the total network length maximized. The heuristic provides for near-optimal solutions to the sensor optimization problem in water distribution networks, with absolute optimality not necessarily sought or being attainable especially under dynamic hydraulic operations.

The proposed approach, which is applicable to longitudinal rather than spatial sensing (thus to devices such as acoustic, pressure, or flow sensors acting on pipe segments), can be readily applied to network topologies of any size and without a prior knowledge of their hydraulic parameters, and can be enhanced by introducing into the analysis greedy-search parameters such as network elevation data. Further, the method does not yield solutions with sensor locations as far as
possible from each other, or with sensors placed along the borders of the area of interest, as Ramakrishnan et al.\cite{13} noted in their work about entropy and mutual information.

Future work on sensor placement and on the proposed entropy-maximization heuristic entails the following actions:

- Expand the method to incorporate a network’s operating parameters further to its topology. This will allow the algorithm to consider real-time operating pressure and flow data, as well as digital elevation models to better optimize sensor locations.
- Use sensor placement optimization to minimize the time to detect water loss or contamination incidents in the piping network.
- Utilize the optimized sensor location to increase the network’s reliability against catastrophic events.

Figure 9. Entropy-based sensor placement optimization (all 8 sensors).

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